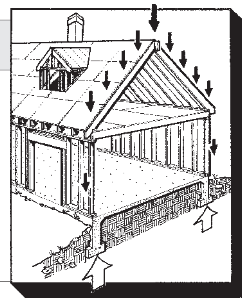

Simple Beam Sizing

by Harris Hyman, P.E.

The fundamental structural element in a building is the beam, a horizontal member that supports something. It's used everywhere in wood-framed structures — joists, headers, girders. In the next two columns, I'd like to show you how we engineers size these beams (headers and girders at least; for joists, you just use the tables). This month we'll go into the raw strength of a beam; next month we'll consider shear, deflection, and bounciness.

All beams, whether wood, steel, concrete, or composite, have five principal properties of interest to the structural engineer: support configuration, cross-section, material, length, and load. From these five properties, we can usually determine whether the beam will do the job.

All these properties apply to the wood members found in a frame house. To simplify things, let's restrict this discussion to "simply supported rectangular wooden beams with evenly distributed loads." What's all this mean? "Simply supported" means that the ends of the beam are resting on two props or supports (see Figure 1). These supports hold the ends up but allow the beam to bend and flex freely. (Other support configurations include cantilevers and beams fixed at one or both ends.) "Rectangular" refers to the section, "wooden" to the material. An "evenly distributed load" is placed along the entire beam.

In the case of floor systems, the "evenly distributed load" is not quite real; it is an ideal used for convenience. Real loads are spotty and moving; they are concentrated where furnishings and people are, and they can move around. Since designers have to set up the floor before knowing where things will be, they use a distributed load — so many pounds per square foot. This is usually a much higher load than would occur in a real building, but it serves for designing.

Simply Supported Beam

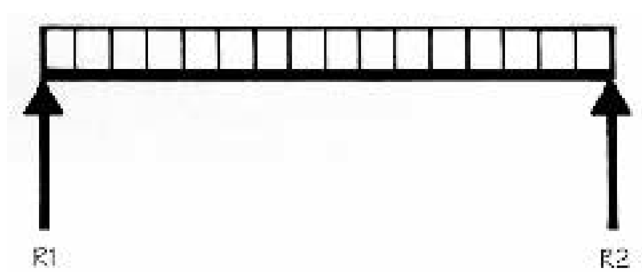

Figure 1. *A "simply supported" beam rests on its supports but is free to flex and bend. Most of the beams in a typical wood frame house — headers and girders, for example — are simply supported beams.*

Tributary Load on a Girder

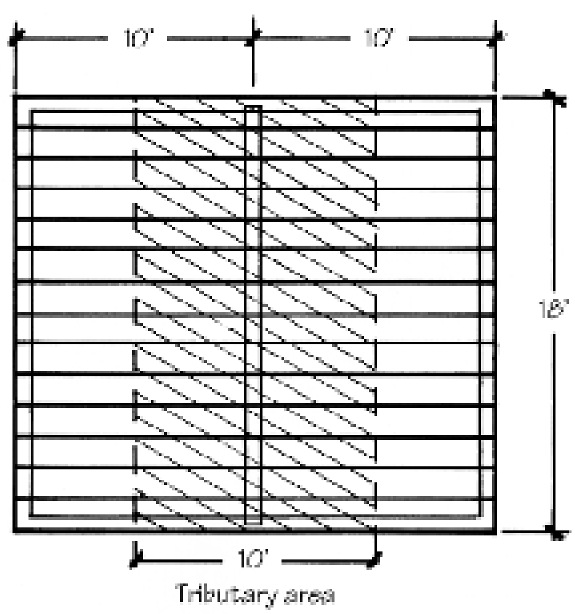

Figure 2. *A typical central girder carries half the load of the joists that rest on it — in this case, an area 10 feet wide by the 18-foot length of the beam.*

A Worked Example

Let's look at a specific beam and see if it does the job. Our arbitrary beam (Figure 2) will be an 18-foot-long girder down the center of a 20-foot-wide living room. The structural system is 10-foot-long 2x8 joists running from the walls to the girder at 16 inches on-center. The joists are #2 Doug fir and the girder is built up of select structural Doug fir 2x12s. In the Uniform Building Code, I find that the floor of a residential building should be designed to support a load of 40 pounds per square foot. We will use all of these specifications to calculate the *bending stress* — the raw strength of the beam.

Stress is the relative amount of load supported by a material. It is measured in pounds per square inch: load divided by area. It is *stress*, not load, that tears things apart. *Bending stress* (sometimes called *extreme fiber stress in bending*) is the stress in the outermost (extreme) fibers of a wood beam, caused by loads perpendicular to the beam.

The symbol for bending stress is *fb*. With a capital "F," *Fb* refers to the *maximum allowable bending stress*, a tested design value that depends on wood species. With a small "f," *fb* refers to the *actual bending stress* induced in a beam by its load. In sizing wood beams, one task is to ensure that *fb* doesn't exceed *Fb*.

Step 1. First, we calculate the load on the beam from the known 40-psf floor load. The floor space that directly loads the girder is called the *tributary area* for the girder — in this case, a 10-foot-wide area down the middle of the floor. At 40 psf, the beam load (w) in pounds per inch is:

$$w = \frac{10 \text{ ft. x } 40 \text{ psf}}{12 \text{ in/ft}}$$

$$= 33.3 \text{ pounds per inch}$$

Because we're working in inches and pounds — the standard English unit of stress for centuries — there is a 12 thrown into the calculation to make the units come out right.

Step 2. Next we calculate a quantity called the *maximum bending moment* (M). This is the tendency for the beam load to stress the beam, and it depends on the beam load, the length of the beam, and the load configuration. For each load configuration there is specific bending moment formula. These formulas are tabulated in various references; I personally use the AISC *Manual of Steel Construction* (American Institute of Steel Construction, One E. Wacker Dr., Suite 3100, Chicago, IL 60601; 312/670-2400). Another source is the AFPA *Wood Structural Design Data* book (American Forest and Paper Association, 1111 19th St., Suite 800, Washington, D.C. 20036; 202/463-2700).

The bending moment formula for a simply supported beam with an evenly distributed load is:

$$M = \frac{(w \text{ x } L^2)}{8}$$

where w is the unit load per inch on the beam and L represents its total length. For our problem beam:

$$M = \frac{33.3 \text{ lb./in. x } (18 \text{ ft. x } 12 \text{ in./ft.})^2}{8}$$

$$= 194{,}206 \text{ in. lb.}$$

So our 18-foot-long girder must be sized to resist a 194,206 in. lb. bending moment.

Step 3. Next we have to select a beam size that we think will be adequate. To check this, we calculate its *section modulus (S)*. Section modulus is a subordinate property of a beam, an expression of its strength in terms of its cross-section (Figure 3). For a rectangular beam, the formula for the section modulus is:

$$S = \frac{b \text{ x } h^2}{6}$$

where b is the width and h is the depth of the member. From the formula you can see what every framer knows intuitively: that a wood member increases in bending strength more in relation to its depth (which is squared) than its width.

We'll need a pretty big girder to span 18 feet — say, a triple 2x12. A single 2x12 has a section modulus:

$$S = \frac{1.5 \text{ x } 11.25^2}{6}$$

$$= 31.6 \text{ in}^3$$

A triple 2x12 is three times this: 94.8 in^3.

No one actually calculates the section modulus for common lumber — we just look up the numbers in a table. You can find one in the National Design Specification supplement, *Design Values for Wood Construction* (also published by AFPA), which I'll excerpt:

Lumber Size	Section Modulus
2x4	3.1
2x6	7.6
2x8	13.1
2x10	21.4
2x12	31.6

Section Modulus (S) of a Rectangular Beam

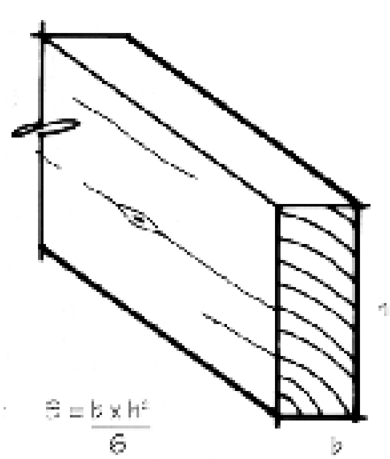

Figure 3. *Section modulus is a property of a beam that engineers use to calculate the bending stress of a given beam under specific loading conditions. The formula shows what every framer knows — that a stick of wood is stronger placed on edge than on its face.*

Step 4. Now we use the section modulus to calculate the *actual bending stress* (fb) that the bending moment will create in the triple 2x12. The relationship between bending moment, section modulus, and bending stress can be expressed in a simple formula:

$$M = fb \text{ x } S$$

Since we know M and S, to solve for fb we set up the equation like this:

$$fb = \frac{M}{S}$$

$$fb = \frac{194{,}206 \text{ in. lb.}}{94.8 \text{ in}^3}$$

$$fb = 2{,}049 \text{ lb./in.}^2 \text{ (psi)}$$

Now we look in the tables of *allowable bending stress* (Fb) for various woods. In *Design Values for Wood Construction* we see that select structural Douglas fir-Larch used as a timber (beams 5x5-inches and larger) has an *allowable* bending stress of 1,600 psi. The *actual* bending stress of 2,049 in the triple 2x12 girder exceeds this by quite a bit, so our first guess was faulty. Let's try a quadruple 2x12, which has a section modulus of 126.4.

We recalculate, using the same bending moment:

$$fb = \frac{194{,}206 \text{ in. lb.}}{126.4}$$

$$= 1{,}536 \text{ psi}$$

So the *actual bending stress* (fb) in the quadruple 2x12 beam — 1,536 psi — is less than the *maximum allowable bending stress* for Doug fir-Larch lumber — 1,600. The beam is strong enough, but we still have to check it for shear and deflection. This we'll do next month. ■

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