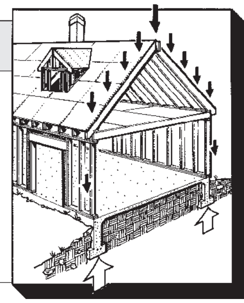


# Down With Sagging Shelves

by Harris Hyman, P.E.

A couple of years back when I moved into my new house and set up my home office, I saw an ad for some attractive bookcases at a good price. They were a Danish import, cleverly designed for easy assembly, and made of particleboard with a hard, white plastic coating. They looked real nice in the store, so I bought several sets.

I hate them. They *were* well made and easy to assemble, and the plastic coating has stood up well and is easy to clean. The fittings are clever and work properly. Everything is as promised. *But the shelves sag!*

Static load deflections are a builder's persistent nightmare. You can craft a home out of elegant materials, but if there are perceptible sags, the whole effect is ruined. Sags are just plain unacceptable, although they have no real effect on the integrity or functionality of the building. And shelves — book, pantry, kitchen, closet — are a common source of these sags.

### Shelf Deflection

Let's analyze my bookcase. The shelves are $8\frac{1}{2}$ inches wide, $\frac{9}{16}$ inch thick, and 26 inches long. *Architectural Graphic Standards* gives the weight of books at 25 pounds per cubic foot. But my shelves are loaded with engineering textbooks, which weigh about 35 pounds per *linear* foot (by an actual weight test). That's the measure I'll use for the rest of this article.

A shelf is — by geometry — a somewhat inefficient beam, laid with the flat side up and thus prone to sag (see "About the Deflection Formula," at right). The *load* ($w$) on the beam is

$$w = \frac{35\ lb./ft.}{12} = 2.9\ lb./in.$$

The deflection ($d$) of this uniformly loaded beam is calculated:

$$d = 0.0130 \times \frac{wL^4}{EI}$$

The *modulus of elasticity* ($E$) of particleboard is about 350,000 psi. The *moment of inertia* ($I$) is calculated:

$$I = \frac{b \times h^3}{12}$$

$$= \frac{8.5 \times 0.56^3}{12}$$

$$= 0.1244\ in.^4$$

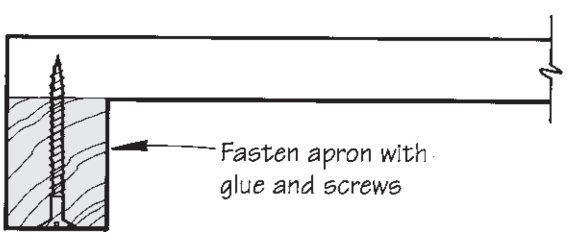
Running these numbers through the deflection formula, you get

$$d = \frac{0.0130 \times 2.9 \times 26^4}{350{,}000 \times 0.1244} = 0.40\ in.$$

The predicted design deflection is about one-third larger than the actual deflection of my shelves, which I measured at 0.29 inch. Perhaps Danish particleboard is stronger or perhaps I've overestimated the weight of my books. Nevertheless, the formula *does* give a plausible estimate for the sag.

***Strength of shelves is not an issue.*** Why not? In answer, take a 1x8 pine

## Shelf Apron




*Adding an apron to the front of a shelf can increase the span by 50% to 75% for a 35-pound-per-linear-foot load. The author recommends attaching the apron with #6 screws and glue.*

## About the Deflection Formula

The basic formula for deflection of beams is

$$d = \frac{K \times wL^4}{EI}$$

where:

$L$ = total span in inches

$w$ = unit load in pounds per inch

$E$ = the *modulus of elasticity*, a measure of the stiffness of the specific beam material, in pounds per square inch

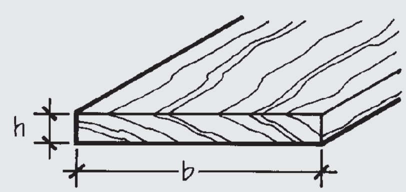
I = moment of inertia, which is a function of the shape and size of beam cross section, measured in inches to the 4th power (in.$^4$).

$K$ is a constant that varies depending on the beam's load configuration (concentrated, distributed, etc.). For simple beams with uniform load, $K$ is 5/384, or .0130, as given in the formula in the article.

The deflection formula is sometimes written with $WL^3$ instead of $wL^4$. It's the same difference: $W$ is the total uniform load along the entire beam; $w$ is the unit load in pounds per inch. Thus, since $w \times L = W$, $wL^4$ is the same as $WL^3$.

The moment of inertia ($I$) is found with the formula

$$I = \frac{b \times h^3}{12}$$




It's easy to see from the formula why a shelf is an inefficient beam. In an ordinary beam, the height ($h$), which is cubed, is the tall dimension of the section. In a shelf, the tall dimension is on its side, while the thickness of the shelf gets cubed.

shelf as an example. The bending stress of this shelf with the 35-pound-per-foot load applied works out to 391 psi. But the *allowable* material bending stress for this shelf is more than 1,000 psi. Under the 35-pound-per-foot-load, all ordinary shelving materials will develop stresses far under the design maximum, and far under breaking stress. Deflection governs the situation; bending strength can be safely ignored.

### How Much Sag Is Okay?

How much deflection is allowable for shelving? I think it's whatever deflection we don't notice without measuring or sighting the line of the shelf. The 0.29-inch deflection of my shelves is *quite* noticeable, with a span-to-deflection ratio of 90, or deflection equal to 1/90th the length of the span. I believe I'd notice half that much deflection (1/180th), but not one-quarter (1/360th). So I'll assume that around $^1/_{32}$ inch per foot (1/384th the length of the span) is an acceptable limit for deflection of shelves.

### Limiting Deflection

So how can do we decrease the deflection of shelves? Look at the deflection formula. We can decrease the load ($w$) or the shelf span ($L$). Or we can increase either the modulus of elasticity ($E$), which is a property of the type material used, or the moment of inertia ($I$), which depends on the width and thickness of the shelf.

But in most cases, the shelf load, $w$, is a basic design parameter and not something we can change. This would assume a lighter load on the shelves, and for careful design, we *must* assume that the owner will pack the shelves full. So that means we must use either shorter or thicker shelves, or a stiffer material. See the table "Allowable Shelf Spans," on page 84, for suggested shelf lengths and sizes. In the table, shelf deflection is limited to 1/400th of the span. I added a column for shelf loads of 60 pounds per linear foot — the expected loading from a stack of canned goods. The additional load shortens the acceptable shelf span by about 15%.

The last lines in the table are for glass. This is an elegant shelf material, though not too useful for a library. Glass is also expensive — about $3.50 per square foot for $^1/_4$ inch and about $11 per square foot for $^1/_2$ inch. Although it's strong, glass is not particularly tough; a shock will destroy it. For safety's sake, make sure any glass you use for shelves is tempered.

Another way to stiffen shelves is to increase $I$ by adding a front apron to the shelf (see illustration, page 80). In most cases, attaching a 1x$^3/_4$-inch apron to the front of the shelf will increase the span by 50% to 75%. Adding a second support at the back of the shelf can add another 25% to the original shelf span. Craftsmanship counts! These aprons must be firmly fastened so that they become an integral part of the shelf. I suggest #6 screws 6 inches on-center and an epoxy glue. ■

## Allowable Shelf Spans

**(deflection limited to 1/400th of span)**

| | Max. length (35-lb. load per linear foot) | Max. length (60-lb. load per linear foot) |
|---|---|---|
| **Softwood (E = 1,200,000 psi)** | | |
| $^1/_2$ x 8 | 18 in. | 15 in. |
| 1 x 8* | 27 | 23 |
| $^5/_4$ x 8* | 38 | 32 |
| $^1/_2$ x 10 | 20 | 16 |
| 1 x 10* | 30 | 25 |
| $^5/_4$ x 10* | 41 | 35 |
| 1 x 12* | 32 | 26 |
| **Hardwood, Doug fir (E = 1,900,000)** | | |
| $^1/_2$ x 8 | 21 | 18 |
| 1 x 8* | 32 | 26 |
| $^5/_4$ x 8* | 44 | 37 |
| $^1/_2$ x 8 | 20 | 16 |
| 1 x 8* | 30 | 25 |
| $^5/_4$ x 8* | 41 | 35 |
| 1 x 12* | 37 | 31 |
| **Particleboard (E = 350,000)** | | |
| $^1/_2$ x 7$^1/_2$ | 12 | 10 |
| $^3/_4$ x 7$^1/_2$ | 18 | 15 |
| **Plywood (E = 1,800,000)** | | |
| $^1/_2$ x 7$^1/_2$ | 17 | 14 |
| $^3/_4$ x 7$^1/_2$ | 26 | 22 |
| **Glass (E = 10,000,000)** | | |
| $^1/_4$ x 4 | 15 | 13 |
| $^1/_2$ x 7$^1/_2$ | 38 | 32 |

* Nominal dimension

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